Problem 1:

Consider a model with one sector, and one type of workers. Individuals can be unemployed or employed. The labor force is constant and normalized to 1. We assume there are u unemployed workers and n employed workers, such that $n + u = 1$. Unemployed workers search for jobs and firms open vacancies to hire them.

We assume that each unemployed worker exerts a search effort e. We denote total job search effort exercised by the unemployed as $u \cdot e$ and total opened vacancies as v. The number of matches resulting from the aggregated search effort and available vacancies is given by the matching function $M(u \cdot e, v)$, which is increasing and concave in both its arguments and homogenous of degree one. The tightness of the labor market is defined as $\theta = \frac{v}{u \cdot e}$. The probability that a vacancy is filled is $q(\theta) = \frac{M(u \cdot e, v)}{v}$. The probability that an unemployed worker exercising one unit of search effort finds a job is $\frac{M(u \cdot e, v)}{ue} = \theta q(\theta) = f(\theta)$.

Unlike in the Diamond-Mortensen-Pissarides (DMP) model with wage bargaining seen in class, we assume that firms pay all workers a fixed wage w (which is above the flow value of unemployment).

Consider firms' decision: All firms are identical, so we can consider for simplicity that there is one representative firm, which generates a product y with its workers $n = 1 - u$. The firm production function is hence equivalent to the aggregate production function, and is represented by a Cobb-Douglas function: $y(n) = a \cdot n^{\alpha}$, with $\alpha \in]0, 1[$ and $a > 0$. Note that this production function implies that there are decreasing returns to labor, unlike in the DMP model where production is assumed proportional to the number of workers. To hire a new worker, the firm posts a vacancy at a cost c. Jobs are exogenously destroyed at a rate s, and r is the interest rate.

(i). Write the Bellman equations for the value of having a vacancy J_v for the firm, and for the value of having a filled job J_e for the firm (in continuous time). Using the free-entry condition, $J_v = 0$, derive formally the equation for firms' labor demand. Provide an interpretation of this equation.

$$rJ_e = \alpha \cdot a \cdot n^{\alpha-1} - w + s(J_v - J_e)$$

Since $J_v = 0$, the two Bellman expressions simplify:

$$J_e = \frac{c}{q(\theta)}$$

$$(r + s)J_e = \alpha \cdot a \cdot n^{\alpha-1} - w$$

So the labor demand can be expressed as:

$$\frac{\alpha \cdot a \cdot n^{\alpha-1}}{(r+s)} = \frac{c}{q(\theta)} + \frac{w}{(r+s)}$$

Firms hire until the marginal gains (the discounted future marginal product of labor $\frac{\alpha \cdot a \cdot n^{\alpha-1}}{(r+s)}$) are equal to the marginal costs (the sum of the recruiting cost $\frac{c}{q(\theta)}$ and the discounted future wage $\frac{w}{(r+s)}$).

Counseling program A large intensive counseling program for unemployed workers is created. When they become unemployed, a fraction β ($\beta \in]0,1[$) of workers receives it, and keeps receiving it as long as they are unemployed. Let's denote u_1 the number of treated unemployed workers (i.e. those receiving the counseling program), and u_0 the number of non-treated unemployed workers, such that $u = u_0 + u_1$. We assume that all unemployed workers keep exerting the same effort e, but the program increases by γ ($\gamma > 1$) the productivity of their search effort, such that we consider that treated workers now exert the effort γe.

(ii). Let's consider the equilibrium of the labor market, with the counseling program. Write down the expression for the time derivatives $\dot{u_0}$ and $\dot{u_1}$. Then, using the fact that at steady state, the numbers of treated unemployed workers and of non-treated unemployed workers are constant, derive the expression for the Beveridge curve:

$$u = \frac{s(\beta/\gamma + 1 - \beta)}{ef(\theta) + s(\beta/\gamma + 1 - \beta)}$$

$$\dot{u_0} = (1-\beta)s(1-u) - e \cdot f(\theta)u_0$$

$$\dot{u_1} = \beta s(1-u) - \gamma \cdot e \cdot f(\theta)u_1$$

At steady state, $\dot{u_0} = \dot{u_1} = 0$, so we have:

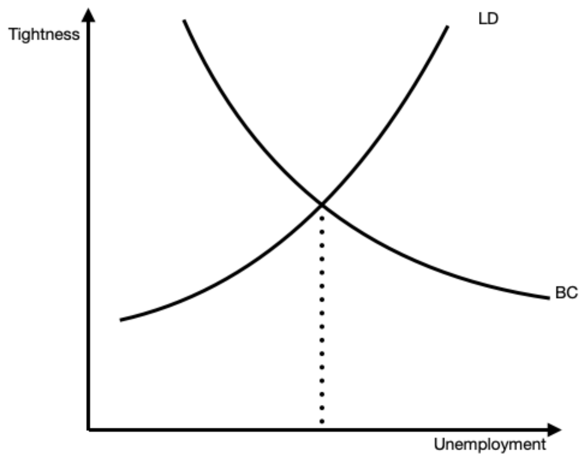
$$u_0 = \frac{(1-\beta)s(1-u)}{e \cdot f(\theta)}$$

$$u_1 = \frac{\beta s(1-u)}{\gamma \cdot e \cdot f(\theta)}$$

From $u = u_0 + u_1$, we can write:

$$u = \frac{(1-\beta)s(1-u)}{e \cdot f(\theta)} + \frac{\beta s(1-u)}{\gamma \cdot e \cdot f(\theta)} \Leftrightarrow$$
$$u = (1-u)\frac{s(\beta/\gamma + 1 - \beta)}{e \cdot f(\theta)} \Leftrightarrow$$
$$u(\frac{e \cdot f(\theta) + s(\beta/\gamma + 1 - \beta)}{e \cdot f(\theta)}) = \frac{s(\beta/\gamma + 1 - \beta)}{e \cdot f(\theta)} \Leftrightarrow$$
$$u = \frac{s(\beta/\gamma + 1 - \beta)}{ef(\theta) + s(\beta/\gamma + 1 - \beta)}$$

Figure 1: Labor market equilibrium, with Cobb-Douglas production function

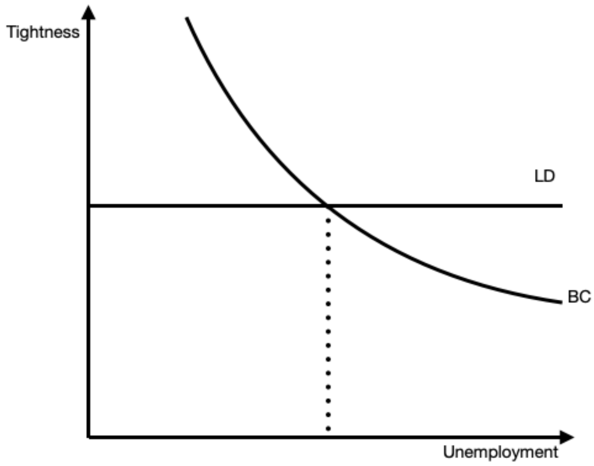


(iii). Consider Figure 1: it represents the Beveridge curve and the labor demand curve in the space (u, θ). The introduction of the counseling program corresponds to an increase in γ, from 1 to a level higher than 1. What happens to the Beveridge curve and to the labor demand curve when γ increases? How does an increase in γ affect the equilibrium level of unemployment, and tightness? You can use graphical representations to explain.

From the expression of the Beveridge curve, we can see that for a given θ, when γ increases, u decreases. From the expression of labor demand, we see that for a given θ, when γ increases, u does not change.
Therefore, the Berveridge curve shifts to the left, but the labor demand curve does not move when γ increases.
The new equilibrium is hence represented graphically in Panel (1) of Solution Figure 1 by the intersection of the BC that is the most to the left, with the labor demand

curve.
It shows that the equilibrium with the counseling program corresponds to a labor market with a lower θ and a lower u. As we assumed that wages are fixed, the equilibrium wage level is constant.

Figure 2: Labor market equilibrium, with proportional production function



(iv). For comparison, we now consider an alternative production function, where production is proportional to the number of workers, like in the DMP model: $y(n) = a \cdot n$, with $a > 0$. Figure 2 represents the corresponding labor demand curve and Beveridge curve in the space (u, θ). Derive the expression of firms' labor demand with this alternative production function. Then discuss what happens in that case to the equilibrium level of unemployment, and tightness, when γ increases. You can use graphical representations to explain.

The labor demand with a proportional production function can be expressed as: $a = \frac{c \cdot (r+s)}{q(\theta)} + w$.
From this expression of labor demand, we see that the labor demand function does not move when γ increases. The effect on the BC is the same as previously.
The new equilibrium is hence represented graphically in Panel (2) of Solution Figure 1 by the intersection of the BC that is the most to the left, with the labor demand curve. It shows that the equilibrium with the counseling program corresponds to a labor market with the same θ but a lower u. As we assumed that wages are fixed, the equilibrium wage level is constant.

(v). Provide intuitions for the differences in the effect of the increase in γ on the equilibrium level of tightness and unemployment when the production function is Cobb-

Douglas (as in question (iii)), and when it is proportional (as in question (iv)).

First, we see that there is no effect on tightness if the production function is proportional, while tightness decreases if it is Cobb-Douglas.
Why? When firms decide whether to create vacancies, they take into account hiring costs, wages (here constant), and the marginal product of labor.

- With a proportional production function, we see that the marginal product of labor does not depend on the number of workers n. Therefore firms' labor demand does not depend on unemployment $1 - u$, beyond the influence of θ on hiring costs. Therefore, for any value of u (or eu), firms maintain tightness constant. When the counseling program pushes search up, tightness goes down, and firms create vacancies until tightness gets back to its prior level.
- In contrast, with the Cobb-Douglas production function, the marginal product of labor decreases when the number of workers increases, and hence when unemployment decreases: $y''(n) < 0$. When the counseling program is introduced, search effort increases, hiring costs decrease, and firms create vacancies, but only up to a point. At some point, they stop creating vacancies even if tightness remains lower (and the cost of hiring remains lower) because the marginal gains from hiring are also lower.

Second, we see that the decrease in the equilibrium level of unemployment is smaller when the production function is Cobb-Douglas. This is because labor market tightness decreased, which means that the competition among unemployed workers to get jobs increased, and therefore it is harder to leave unemployment.

Note that there is no effect on equilibrium wages in both cases, by assumption.

(vi). For the rest of the problem, we always assume the production function is Cobb-Douglas, as assumed initially. Even for non-treated workers, the job finding rate is affected by the counseling program. Explain why with words.

The non-treated jobseekers keep the same search effort with the counselling program, but they face a lower labor market tightness and $f'(\theta) > 0$. Therefore, their job finding rate decreases.

(vii). Assume the workers who receive the counseling program are selected randomly. Can we obtain a non biased estimate of the causal effect of the program on unemployment duration, by comparing the unemployment duration of treated workers to the unemployment duration of non-treated workers? Explain with words.

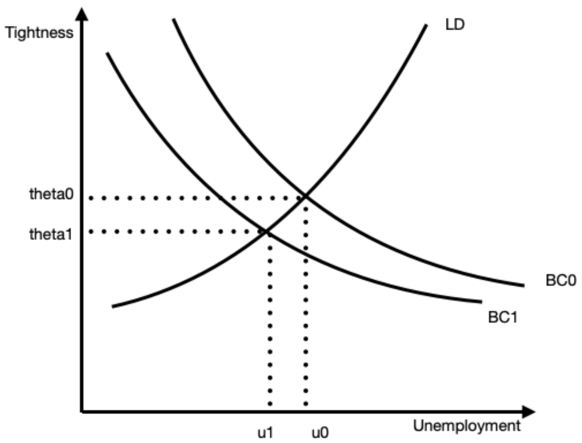
No. The causal effect of the program on its recipients ("microeconomic effect") is the difference between the outcomes for the treated individuals and their counterfactual outcome *if the counseling program did not exist*. This counterfactual outcome is different from the outcome of non-treated individuals. Indeed, we have shown that non-treated job seekers are indirectly affected by the counseling program: their job finding is different when the program is implemented. In other words, the program generates externalities on non-treated. If we compare treated and non-treated individuals, we will obtain a biased estimate of the causal effect, because it will not account for these externalities.

Note that if the fraction of treated individuals β is small, the bias will be small.

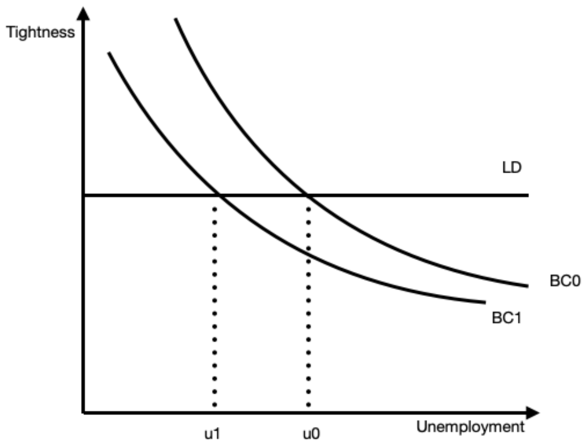
Note as well that, because the treatment is randomized, the treated group would be comparable to the non-treated group, even if there was heterogeneity in the population of workers (in this exercise, we assumed that all workers are identical anyway). In the absence of externality, comparing the treated and the non-treated group could therefore give the causal micro-economic effect of the program.

Solution Figure 1: Labor market equilibrium, when γ increases

(1) Cobb-Douglas production function



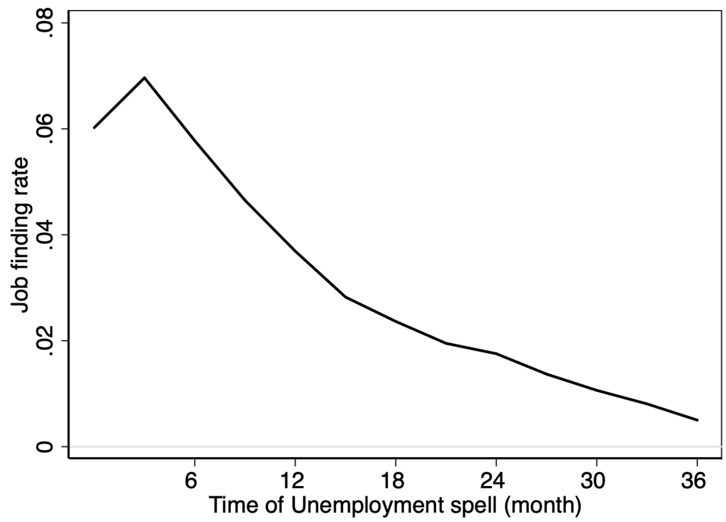
(2) Proportional production function



Problem 2:

(i). In many countries, we observe that the rate of job finding among unemployed workers decreases over time, like in Figure 3. It does not necessarily mean that the probability of finding a job for unemployed workers decreases over time. Explain with words what could be an alternative factor. Then suggest one empirical method that would help neutralize this alternative factor and estimate how the probability of finding a job for unemployed workers changes over time (several possible answers).

Figure 3: Job finding rate and time of unemployment



Notes: This figure shows the rate of unemployed workers who found a job each month, after different duration of unemployment in France.

This pattern could reflect dynamic selection: assume there are two types of unemployed workers that lose their job in the same time: one with a *constant* and high job finding probability and the other with a *constant* and low job finding probability. Over time, some workers will find a job and leave the population of unemployed workers, but more from the high type will leave than from the low type. The proportion of the low type will increase among workers still unemployed. Therefore, the job finding rate will decrease.

To test if the probability of finding a job might decrease with time in unemployment, Kroft et al. (2013) have conducted an audit study: they submitted fictitious resumes to real, online job postings, and tracked callbacks from employers for each submission. In designing each resume, they randomized the length of the current unemployment spell from 1 to 36 months, so that the unemployment spell length is orthogonal to all of the other characteristics of the resume that are observable by potential employers. With their design, the authors made sure that the fictitious unemployed workers with different unemployment spells are similar in their other characteristics.

Any difference in the rate of call back cannot be explained by differences in their other characteristics (dynamic selection), but by the causal effect on unemployment duration on the probability of a call back. They found a negative relation between unemployment duration and the callback probability. This result hence shows that the unemployed workers experience a lower and lower probability of call back over time.

Other empirical methods seen in class can be used to try to address the identification problem posed by dynamic selection:

- Controlling for observable characteristics
- Using a mixed proportional hazard model (Lalive et al. (2006))
- (more indirectly) Collecting longitudinal data on job search behavior, to document within-individual changes in behavior over time (Marinescu and Skandalis (2021))

First, let's consider the situation of a group of unemployed workers in a stationary environment. Assume that all individuals are identical in this group. They receive benefits b when they are unemployed and search for a job with a fixed and costless search effort (assume it is 1). Search is random: job seekers draw offers from the wage offer distribution $H(.)$, they receive a job offer with a probability λ, and decide if they reject or accept the job offer. When employed, workers do not search. Jobs are exogenously destroyed at a rate q, and r is the interest rate.

(ii). Write the Bellman equations for value of being employed, the value of being unemployed, and derive the expression of the reservation wage.

Bellman equation for the value of being employed:

$$rV_e(w) = w + q(V_u - V_e(w)) \tag{1}$$

Bellman equation for the value of being unemployed:

$$rV_u = b + \lambda \int_x^{+\infty} [V_e(w) - V_u] dH(w) \tag{2}$$

We rewrite (1) as $V_e(w) - V_u = \frac{w - rV_u}{r+q}$, and plug it in (2), such that:

$$rV_u = b + \lambda \int_x^{+\infty} [\frac{w - rV_u}{r + q}] dH(w)$$

The reservation wage is the wage x that makes workers indifferent between being unemployed or employed: $V_u = V_e(x)$ and therefore $rV_u = rV_e(x) = x$.

So :

$$x = b + \frac{\lambda}{r+q} \int_x^{+\infty} [w - x] dH(w)$$

(iii). How does the reservation wage change when λ increases? Show it formally, and then provide some intuition.

We take the derivative of the expression for the reservation wage, it gives:

$$\frac{dx}{d\lambda} = \frac{1}{r+q} \int_x^{+\infty} [w-x] dH(w) + \frac{\lambda}{r+q} \frac{dx}{d\lambda} \frac{d}{dx} \left[\int_x^{+\infty} [w-x] dH(w) \right]$$

We use the Leibniz integral rule:

$$\frac{d}{dx} \left(\int_x^b f(x,t) dt \right) = -f(x,x) + \int_x^b \frac{\partial}{\partial x} f(x,t) dt$$

We obtain:

$$\frac{d}{dx} \left[\int_x^{+\infty} [w-x] dH(w) \right] = -(1 - H(x))$$

We insert this in the expression for derivative of the reservation wage with respect to λ, and re-arrange:

$$\frac{dx}{d\lambda} (r + q + \lambda(1 - H(x))) = \int_x^{+\infty} [w-x] dH(w)$$

$$\frac{dx}{d\lambda} = \frac{\int_x^{+\infty} [w-x] dH(w)}{r + q + \lambda(1 - H(x))} > 0$$

Therefore, the reservation wage increases when the job offer arrival increases. The value of staying unemployed increases, as each period in unemployment sees more new job offers coming. Unemployed workers can therefore be more selective with the offer they accept.

Now we consider that the environment of workers is non-stationary. We assume that the probability of receiving an offer is a function of the time spent unemployed $\lambda(t)$ and drops after one year unemployed. $\lambda(t)$ hence takes 2 values: λ_1 before 1 year, and λ_2 after, with $0 < \lambda_2 < \lambda_1 < 1$.

(iv). Derive the expression of workers' reservation wage after one year. Then, based on the differential equation of reservation wages and on formal arguments, explain how workers' reservation wages evolve during the first year.

When $t > 1$ year, workers are in a stationary environment, given that their job arrival rate will not change anymore. Therefore, the expression of their reservation wage is the same as the one derived in question (ii).

$$x(t) = x_2 = b + \frac{\lambda_2}{r+q}\int_{x_2}^{+\infty}[w - x_2]dH(w)$$

When $t \leq 1$ year, workers are in a non-stationary environment. Their value of being unemployed can be written:

$$rV_u(t) = b + \lambda_1 \int_{x}^{+\infty}[\frac{w - rV_u(t)}{r+q}]dH(w) + \dot{V_u}(t)$$

And therefore their reservation wage:

$$x(t) = b + \lambda_1 \int_{x(t)}^{+\infty}[\frac{w - x(t)}{r+q}]dH(w) + \frac{\dot{x(t)}}{r}$$

That gives us the following differential equation for the reservation wage before 1 year:

$$\dot{x}(t) = rx(t) - rb - r\lambda_1 \int_{x(t)}^{+\infty}[\frac{w - x(t)}{r+q}]dH(w)$$

What does this differential equation on the evolution of reservation wage before 1 year? We first note that $\dot{x}(t)$ is strictly increasing in $x(t)$, as:

$$\frac{\partial \dot{x}(t)}{\partial x(t)} = r + \frac{r\lambda_1}{r+q}(1 - H(x(t))) > 0$$

We then define x_1, such that $x(t) = x_1$ if $\lambda(t) = \lambda_1$ and $\dot{x}(t) = 0$. We can see that:

$$x_1 = b + \frac{\lambda_1}{r+q}\int_{x_1}^{+\infty}(w - x_1)\,dH(w)$$

We showed in question (iii) that $\frac{\partial x}{\partial \lambda} > 0$. Therefore $\lambda_2 < \lambda_1$ implies that $x_2 < x_1$

What is the value of reservation wage in the first period of unemployment $x(0)$? As $\dot{x}(t)$ is strictly increasing in $x(t)$, and $x(t) = x_1$ for $\dot{x}(t) = 0$, the reservation wage must decrease if it starts from an initial value $x(0)$ below x_1, and increase if it starts above x_1 (as illustrated in Solution Figure 2).

- If $x(0) < x_2$, $x(t)$ would decrease below x_2, and diverge from x_2
- If $x(0) > x_1$, $x(t)$ would increase above x_1, and diverge from x_2
- We must have: $x_2 < x(0) < x_1$. Therefore, x decreases over time until it gets to x_2 after 1 year.

(v). We still assume that the job arrival rate is as described in question (iv). But now, we assume that some workers never realize that their job arrival rate drops after 1 year: at all periods, they make their decision as if, for all t, $\lambda(t) = \lambda_1$. Let's denote α the type of workers who never realizes that the job arrival rate drops, and β the type of workers who knows and behaves accordingly (like described in question (iv)). Which group of workers has a higher hazard of finding a job? Which group of workers has higher expected re-employment wages? Show it formally, and then provide some intuition.

Let's first look at the hazard rate. As workers α don't know that their job arrival rate changes after 1 year, they have the same reservation wage as for a constant λ_1:

$$x^{\alpha}(t) = x_1 = b + \frac{\lambda_1}{r+q} \int_{x_1}^{+\infty} [w - x_1] dH(w)$$

Therefore, the hazard of finding a job for workers of type α:

- Before one year is: $\phi^{\alpha}(t) = \lambda_1 \left(1 - H\left(x_1\right)\right)$
- After one year is: $\phi^{\alpha}(t) = \lambda_2 \left(1 - H\left(x_1\right)\right)$

For workers β, the reservation wage is as described in question (iv): Before 1 year, $x_2 < x^{\beta}(t) < x_1$, and after 1 year, $x^{\beta}(t) = x_2$. Therefore, the hazard of finding a job for workers of type β:

- Before one year is: $\phi^{\beta}(t) = \lambda_1 \left(1 - H\left(x^{\beta}(t)\right)\right)$
- After one year is: $\phi^{\beta}(t) = \lambda_2 \left(1 - H\left(x_2\right)\right)$

As $\frac{\partial \phi}{\partial x} < 0$, the workers β will have a higher hazard of finding a job in all periods.

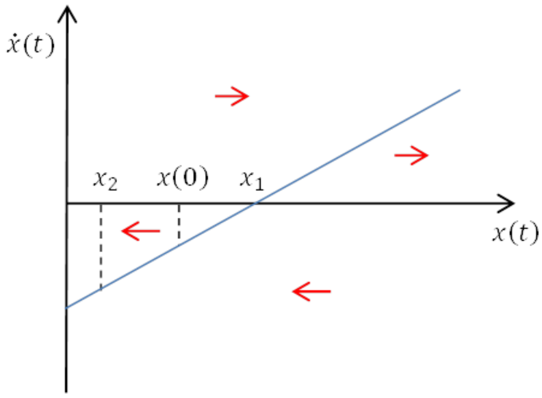
Now, let's consider the re-employment wages:

$$E\left(w | w \geq x\right) = \frac{\int_{x}^{+\infty} w dH(w)}{1 - H(x)}$$

As $\frac{\partial E(w|w \geq x)}{\partial x} > 0$, the workers β will have a lower expected re-employment wage job in all periods.

Intuitively, when workers know about the drop in their job arrival rate, they realize that the value of staying unemployed will be lower in the future, and that their chance to get an attractive offer in the future is low. Therefore, they lower their reservation wage to try to leave unemployment sooner. That increases their hazard of finding a job. Conversely, the workers who don't realize that the job arrival rate drops, keep a relatively high reservation wage, as they expect that staying unemployed will bring them attractive job offers in the near future. Their hazard of finding a job is lower. However, those who are lucky and draw a job offer above their (relatively high) reservation wage, go back to work with a high wage.

Solution Figure 2: Evolution of reservation wage before 1 year



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